Mathematical Proofs

Chapter 4 – Direct Proof and Proof by Contrapositive (Exercise solutions)

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## Section 1: Trivial and Vacuous Proofs

### Exercises

1. Let . Prove that if , then
   1. Since , it follows that for all . Hence the statement is true trivially.
2. Let . Prove that if , then .
   1. Since and , it follows that and the statement is false for all . Hence the statement is true vacuously.
3. Let . Prove that if , then .
   1. Note that . If , then ; while if , then and so . Thus is false for all and so the statement is true vacuously.
4. Let . Prove that if , then .
   1. Note that . Since , it follows that and so the statement is true trivially.
5. Let . Prove that if , then .
   1. Since , it follows that and so . Thus the statement is true vacuously.
6. Prove that if a, b and c are odd integers such that , then . (You are permitted to use well-known properties of integers here.)
   1. Since the sum of any two odd integers is always even, and the sum of an even and an odd integer is always odd, the sum of will always be odd. Hence is always false. Thus the statement is true vacuously.
7. Prove that if are three real numbers such that , then
   1. Since , it follows that and so . Thus the statement is true vacuously.

## Section 2: Direct Proofs

### Exercises

1. Prove that if x is an odd integer, then is even.
   1. Assume that x is an odd integer. Since x is odd, we can write for some integer n. Now . Since is an integer, is even.
2. Prove that if x is an even integer, then is an odd integer.
   1. Assume that x is an even integer. Since x is even, we can write for some integer n. Now . Since is an integer, is odd.
3. Prove that if a and c are odd integers, then is even for every integer b.
   1. Assume a and c are odd integers. Observe that if b is even, and will be even, and thus will be even. If b is odd, and will be odd, and thus will also be even.
4. Let . Prove that if , then is an even integer.
   1. Assume that . Then . Thus is an even integer.
5. Let . Prove that if is an odd integer, then is an odd integer.
   1. Assume is odd. Then . Thus is an odd integer.
6. Let and let . Prove that if is even, then is even.
   1. Assume that is even. Then and, which is even.
7. Let . Prove that if and is odd, then is even.
   1. Assume and is odd. Then and . Thus the statement is false!
8. Let and . Prove that if is prime.
   1. Assume that . Then are all primes.